4. For several values of $x$, use MATLAB to confirm that $\sinh x = (e^x - e^{-x})/2$.

5. For several values of $x$, use MATLAB to confirm that $\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1}), -\infty < x < \infty$.

6. The capacitance of two parallel conductors of length $L$ and radius $r$, separated by a distance $d$ in air, is given by

$$C = \frac{\pi \epsilon L}{\ln\left(\frac{d-r}{r}\right)}$$

where $\epsilon$ is the permittivity of air ($\epsilon = 8.854 \times 10^{-12}$ F/m).

Write a script file that accepts user input for $d$, $L$, and $r$, and computes and displays $C$. Test the file with the values: $L = 1$ m, $r = 0.001$ m, and $d = 0.004$ m.

7.* When a belt is wrapped around a cylinder, the relation between the belt forces on each side of the cylinder is

$$F_1 = F_2 e^{\mu\beta}$$

where $\beta$ is the angle of wrap of the belt and $\mu$ is the friction coefficient. Write a script file that first prompts a user to specify $\beta$, $\mu$, and $F_2$ and then computes the force $F_1$. Test your program with the values $\beta = 130°$, $\mu = 0.3$, and $F_2 = 100$ N. (Hint: Be careful with $\beta$!)

## Section 3.2

8. The MATLAB trigonometric functions expect their argument to be in radians. Write a function called `sind` that accepts an angle $x$ in degrees and computes $\sin x$. Test your function.

9. Write a function that accepts temperature in degrees F and computes the corresponding value in degrees C. The relation between the two is

$$T\ °\text{C} = \frac{5}{9}(T\ °\text{F} - 32)$$

Be sure to test your function.

10.* An object thrown vertically with a speed $v_0$ reaches a height $h$ at time $t$, where

$$h = v_0 t - \frac{1}{2} g t^2$$

Write and test a function that computes the time $t$ required to reach a specified height $h$, for a given value of $v_0$. The function's inputs should be $h$, $v_0$, and $g$. Test your function for the case where $h = 100$ m, $v_0 = 50$ m/s, and $g = 9.81$ m/s$^2$. Interpret both answers.

11. A water tank consists of a cylindrical part of radius $r$ and height $h$, and a hemispherical top. The tank is to be constructed to hold 500 m$^3$ when filled. The surface area of the cylindrical part is $2\pi r h$, and its volume is $\pi r^2 h$. The surface area of the hemispherical top is given by $2\pi r^2$, and its

volume is given by $2\pi r^3/3$. The cost to construct the cylindrical part of the tank is \$300 per square meter of surface area; the hemispherical part costs \$400 per square meter. Use the `fminbnd` function to compute the radius that results in the least cost. Compute the corresponding height $h$.

12. A fence around a field is shaped as shown in Figure P12. It consists of a rectangle of length $L$ and width $W$, and a right triangle that is symmetrical about the central horizontal axis of the rectangle. Suppose the width $W$ is known (in meters), and the enclosed area $A$ is known (in square meters). Write a user-defined function file with $W$ and $A$ as inputs. The outputs are the length $L$ required so that the enclosed area is $A$, and the total length of fence required. Test your function for the values $W = 6$ m and $A = 80\ \text{m}^2$.

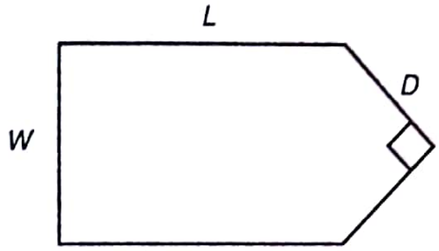


**Figure P12**

13. A fenced enclosure consists of a rectangle of length $L$ and width $2R$, and a semicircle of radius $R$, as shown in Figure P13. The enclosure is to be built to have an area $A$ of 1600 ft$^2$. The cost of the fence is \$40 per foot for the curved portion, and \$30 per foot for the straight sides. Use the `fminbnd` function to determine with a resolution of 0.01 ft the values of $R$ and $L$ required to minimize the total cost of the fence. Also compute the minimum cost.

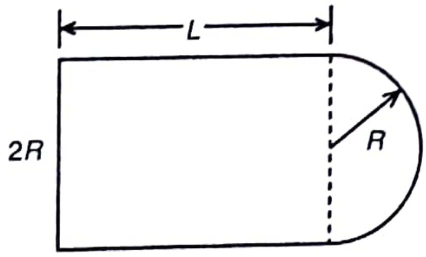


**Figure P13**

14. Using estimates of rainfall, evaporation, and water consumption, the town engineer developed the following model of the water volume in the reservoir as a function of time.

$$V(t) = 10^9 + 10^8(1 - e^{-t/100}) - rt$$

where $V$ is the water volume in liters, $t$ is time in days, and $r$ is the town's consumption rate in liters/day. Write two user-defined functions. The first function should define the function $V(t)$ for use with the `fzero` function.

The second function should use `fzero` to compute how long it will take for the water volume to decrease to $x$ percent of its initial value of $10^9$ L. The inputs to the second function should be $x$ and $r$. Test your functions for the case where $x = 50$ percent and $r = 10^7$ L/day.

15. The volume $V$ and paper surface area $A$ of a conical paper cup are given by

$$V = \frac{1}{3}\pi r^2 h \qquad A = \pi r \sqrt{r^2 + h^2}$$

where $r$ is the radius of the base of the cone and $h$ is the height of the cone.

*a.* By eliminating $h$, obtain the expression for $A$ as a function of $r$ and $V$.

*b.* Create a user-defined function that accepts $R$ as the only argument and computes $A$ for a given value of $V$. Declare $V$ to be global within the function.

*c.* For $V = 10$ in.$^3$, use the function with the `fminbnd` function to compute the value of $r$ that minimizes the area $A$. What is the corresponding value of the height $h$? Investigate the sensitivity of the solution by plotting $V$ versus $r$. How much can $R$ vary about its optimal value before the area increases 10 percent above its minimum value?

16. A torus is a shaped like a doughnut. If its inner radius is $a$ and its outer radius is $b$, its volume and surface area are given by

$$V = \frac{1}{4}\pi^2(a + b)(b - a)^2 \qquad A = \pi^2(b^2 - a^2)$$

*a.* Create a user-defined function that computes $V$ and $A$ from the arguments $a$ and $b$.

*b.* Suppose that the outer radius is constrained to be 2 in. greater than the inner radius. Write a script file that uses your function to plot $A$ and $V$ versus $a$ for $0.25 \le a \le 4$ in.

17. Suppose it is known that the graph of the function $y = ax^3 + bx^2 + cx + d$ passes through four given points $(x_i, y_i)$, $i = 1, 2, 3, 4$. Write a user-defined function that accepts these four points as input and computes the coefficients $a$, $b$, $c$, and $d$. The function should solve four linear equations in terms of the four unknowns $a$, $b$, $c$, and $d$. Test your function for the case where $(x_i, y_i) = (-2, -20)$, $(0, 4)$, $(2, 68)$, and $(4, 508)$, whose answer is $a = 7$, $b = 5$, $c = -6$, and $d = 4$.

## Section 3.3

18. Use the `gen_plot` function described in Section 3.3 to obtain two subplots, one plot of the function $10e^{-2x}$ over the range $0 \le x \le 2$, and the other a plot of $5\sin(2\pi x/3)$ over the range $0 \le x \le 6$.

19. Create an anonymous function for $10e^{-2x}$ and use it to plot the function over the range $0 \le x \le 2$.

**15.** Are these following expressions equivalent? Use MATLAB to check your answer for specific values of *a*, *b*, *c*, and *d*.

*a.* 1. `(a==b)&((b==c)|(a==c))`
   2. `(a==b)|((b==c)&(a==c))`

*b.* 1. `(a<b)&((a>c)|(a>d))`
   2. `(a<b)&(a>c)|((a<b)&(a>d))`

## Section 4.4

**16.** Rewrite the following statements to use only one `if` statement.

```
if x < y
    if z < 10
        w = x*y*z
    end
end
```

**17.** Write a program that accepts a numerical value *x* from 0 to 100 as input and computes and displays the corresponding letter grade given by the following table.

A $x \geq 90$
B $80 \leq x \leq 89$
C $70 \leq x \leq 79$
D $60 \leq x \leq 69$
F $x < 60$

*a.* Use nested `if` statements in your program (do not use `elseif`).
*b.* Use only `elseif` clauses in your program.

**18.** Write a program that accepts a year and determines whether or not the year is a leap year. Use the `mod` function. The output should be the variable `extra_day`, which should be 1 if the year is a leap year and 0 otherwise. The rules for determining leap years in the Gregorian calendar are:

1. All years evenly divisible by 400 are leap years.
2. Years evenly divisible by 100 but not by 400 are not leap years.
3. Years divisible by 4 but not by 100 are leap years.
4. All other years are not leap years.

For example, the years 1800, 1900, 2100, 2300, and 2500 are not leap years, but 2400 is a leap year.

**19.** Figure P19a shows a mass-spring model of the type used to design packaging systems and vehicle suspensions, for example. The springs exert a force that is proportional to their compression, and the proportionality constant is the spring constant *k*. The two side springs provide additional resistance if the weight *W* is too heavy for the center spring. When the weight *W* is gently placed, it moves through a distance *x* before coming to rest. From statics, the weight force must balance the spring forces at this new position. Thus

$$W = k_1 x \qquad \text{if } x < d$$
$$W = k_1 x + 2k_2(x - d) \qquad \text{if } x \geq d$$

These relations can be used to generate the plot of $W$ versus $x$, shown in Figure P19b.

*a.* Create a function file that computes the distance $x$, using the input parameters $W$, $k_1$, $k_2$, and $d$. Test your function for the following two cases, using the values $k_1 = 10^4$ N/m; $k_2 = 1.5 \times 10^4$ N/m; $d = 0.1$ m.

$$W = 500 \text{ N}$$
$$W = 2000 \text{ N}$$

*b.* Use your function to plot $x$ versus $W$ for $0 \leq W \leq 3000$ N for the values of $k_1$, $k_2$, and $d$ given in part *a*.

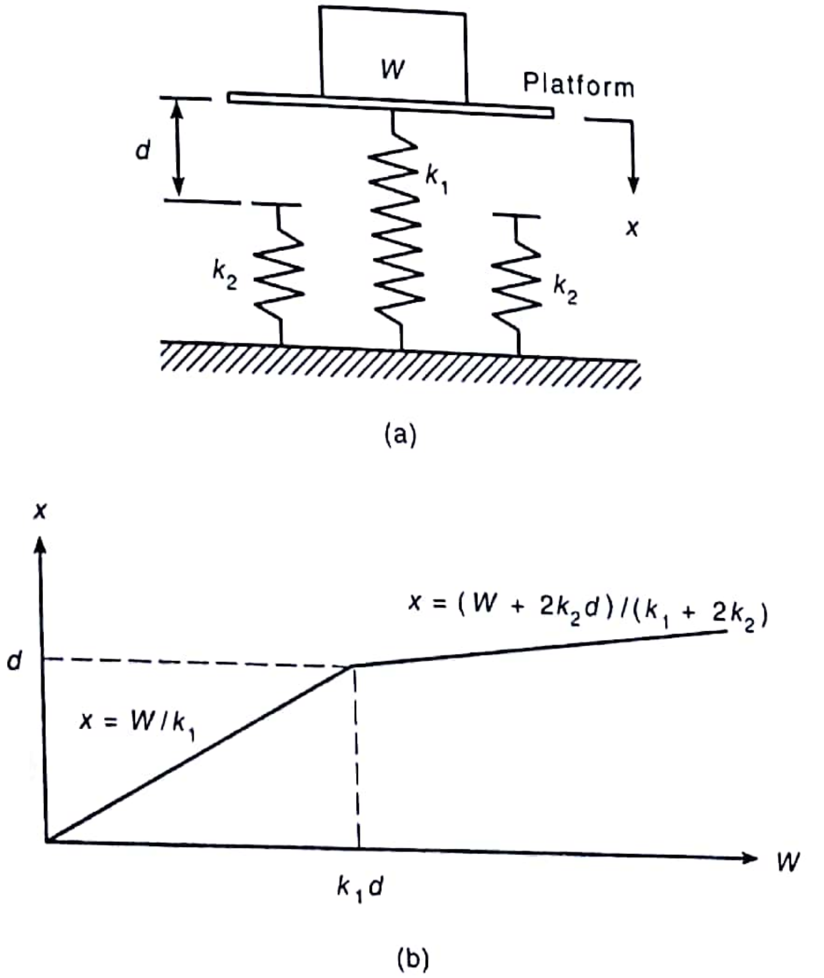


Figure P19

## Section 4.5

**20.** The $(x, y)$ coordinates of a certain object as a function of time $t$ are given by

$$x(t) = 5t - 10 \qquad y(t) = 25t^2 - 120t + 144$$

for $0 \le t \le 4$. Write a program to determine the time at which the object is the closest to the origin at (0, 0). Determine also the minimum distance. Do this two ways:

*a.* By using a `for` loop.
*b.* By not using a `for` loop.

**21.** Consider the array **A**.

$$\mathbf{A} = \begin{bmatrix} 3 & 5 & -4 \\ -8 & -1 & 33 \\ -17 & 6 & -9 \end{bmatrix}$$

Write a program that computes the array **B** by computing the natural logarithm of all the elements of **A** whose value is no less than 1, and adding 20 to each element that is equal to or greater than 1. Do this two ways:

*a.* By using a `for` loop with conditional statements.
*b.* By using a logical array as a mask.

**22.** We want to analyze the mass-spring system discussed in Problem 19 for the case in which the weight $W$ is dropped onto the platform attached to the center spring. If the weight is dropped from a height $h$ above the platform, we can find the maximum spring compression $x$ by equating the weight's gravitational potential energy $W(h + x)$ with the potential energy stored in the springs. Thus

$$W(h + x) = \tfrac{1}{2}k_1x^2 \qquad \text{if } x < d$$

which can be solved for $x$ as

$$x = \frac{W \pm \sqrt{W^2 + 2k_1Wh}}{k_1} \qquad \text{if } x < d$$

and

$$W(h + x) = \tfrac{1}{2}k_1x^2 + \tfrac{1}{2}(2k_2)(x - d)^2 \qquad \text{if } x \ge d$$

which gives the following quadratic equation to solve for $x$:

$$(k_1 + 2k_2)x^2 - (4k_2d + 2W)x + 2k_2d^2 - 2Wh = 0 \qquad \text{if } x \ge d$$

*a.* Create a function file that computes the maximum compression $x$ due to the falling weight. The function's input parameters are $k_1$, $k_2$, $d$, $W$, and $h$. Test your function for the following two cases, using the values $k_1 = 10^4$ N/m; $k_2 = 1.5 \times 10^4$ N/m; and $d = 0.1$ m.

$$W = 100 \text{ N}, h = 0.5 \text{ m}$$

$$W = 2000 \text{ N}, h = 0.5 \text{ m}$$

*b.* Use your function file to generate a plot of $x$ versus $h$ for $0 \le h \le 2$ m. Use $W = 100$ N and the preceding values for $k_1$, $k_2$, and $d$.

**23.** Electrical resistors are said to be connected "in series" if the same current passes through each and "in parallel" if the same voltage is applied across each. If in series, they are equivalent to a single resistor whose resistance is given by

$$R = R_1 + R_2 + R_3 + \cdots + R_n$$

If in parallel, their equivalent resistance is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$$

Write an M-file that prompts the user for the type of connection (series or parallel) and the number of resistors $n$ and then computes the equivalent resistance.

**24.** *a.* An *ideal* diode blocks the flow of current in the direction opposite that of the diode's arrow symbol. It can be used to make a *half-wave rectifier* as shown in Figure P24a. For the ideal diode, the voltage $v_L$ across the load $R_L$ is given by

$$v_L = \begin{cases} v_S & \text{if } v_S > 0 \\ 0 & \text{if } v_S \le 0 \end{cases}$$

Suppose the supply voltage is

$$v_S(t) = 3e^{-t/3} \sin(\pi t) \text{ volts}$$

where time $t$ is in seconds. Write a MATLAB program to plot the voltage $v_L$ versus $t$ for $0 \le t \le 10$.

*b.* A more accurate model of the diode's behavior is given by the *offset diode* model, which accounts for the offset voltage inherent in semiconductor diodes. The offset model contains an ideal diode and a battery whose voltage equals the offset voltage (which is approximately 0.6 V for silicon diodes) [Rizzoni, 1996]. The half-wave rectifier using this model is shown in Figure P24b. For this circuit,

$$v_L = \begin{cases} v_S - 0.6 & \text{if } v_S > 0.6 \\ 0 & \text{if } v_S \le 0.6 \end{cases}$$

Using the same supply voltage given in part *a*, plot the voltage $v_L$ versus $t$ for $0 \le t \le 10$; then compare the results with the plot obtained in part *a*.

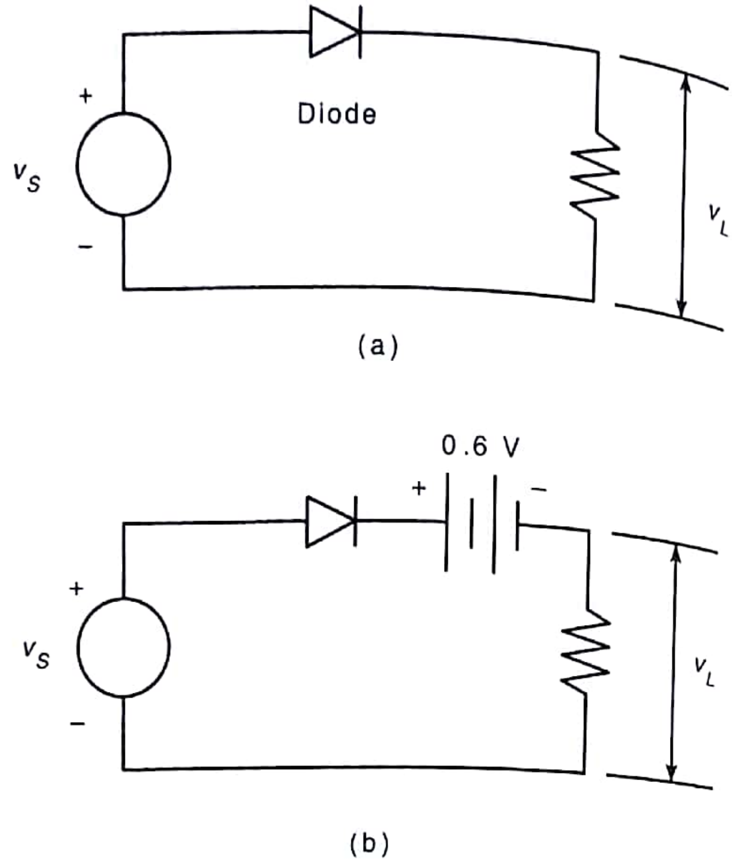


Figure P24

**25.*** Engineers in industry must continually look for ways to make their designs and operations more efficient. One tool for doing so is *optimization*, which uses a mathematical description of the design or operation to select the best values of certain variables. Many sophisticated mathematical tools have been developed for this purpose, and some are in the MATLAB Optimization toolbox. However, problems that have a limited number of possible variable values can use MATLAB loop structures to search for the optimum solution. This problem and the next two are examples of multivariable optimization that can be done with the basic MATLAB program.

A company wants to locate a distribution center that will serve six of its major customers in a 30 × 30 mi area. The locations of the customers relative to the southwest corner of the area are given in the following table in terms of $(x, y)$ coordinates (the $x$ direction is east; the $y$ direction is north) (see Figure P25). Also given is the volume in tons per week that must be delivered from the distribution center to each customer. The weekly delivery cost $c_i$ for customer $i$ depends on the volume $V_i$ and the distance $d_i$ from the distribution center. For simplicity we will assume that this distance is the straight-line distance. (This assumes that

| | Product | | | | Hours available |
|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | |
| Hours required | | | | | |
| Lathe | 1 | 2 | 0.5 | 3 | 40 |
| Grinder | 0 | 2 | 4 | 1 | 30 |
| Milling | 3 | 1 | 5 | 2 | 45 |
| Unit profit ($) | 100 | 150 | 90 | 120 | |

*a.* Determine how many units of each product the company should make to maximize its total profit and then compute this profit. Remember, the company cannot make fractional units, so your answer must be in integers. (Hint: First estimate the upper limits on the number of products that can be produced without exceeding the available capacity.)

*b.* How sensitive is your answer? How much does the profit decrease if you make one more or one less item than the optimum?

**27.** A certain company makes televisions, stereo units, and speakers. Its parts inventory includes chassis, picture tubes, speaker cones, power supplies, and electronics. The inventory, required components, and profit for each product appear in the following table. Determine how many of each product to make in order to maximize the profit.

| | Product | | | |
|---|---|---|---|---|
| | Television | Stereo unit | Speaker unit | Inventory |
| Requirements | | | | |
| Chassis | 1 | 1 | 0 | 450 |
| Picture Tube | 1 | 0 | 0 | 250 |
| Speaker Cone | 2 | 2 | 1 | 800 |
| Power Supply | 1 | 1 | 0 | 450 |
| Electronics | 2 | 2 | 1 | 600 |
| Unit profit ($) | 80 | 50 | 40 | |

**28.*** Use a loop in MATLAB to determine how long it will take to accumulate $1,000,000 in a bank account if you deposit $10,000 initially and $10,000 at the end of each year; the account pays 6 percent annual interest.

**29.** A weight $W$ is supported by two cables anchored a distance $D$ apart (see Figure P29). The cable length $L_{AB}$ is given, but the length $L_{AC}$ is to be selected. Each cable can support a maximum tension force equal to $W$. For the weight to remain stationary, the total horizontal force and total vertical force must each be zero. This principle gives the equations

$$-T_{AB}\cos\theta + T_{AC}\cos\phi = 0$$

$$T_{AB}\sin\theta + T_{AC}\sin\phi = W$$

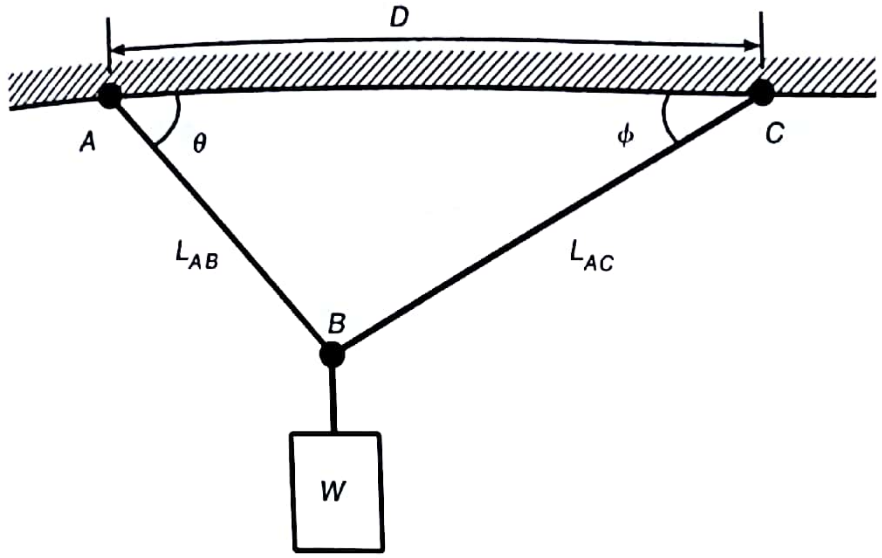


Figure P29

We can solve these equations for the tension forces $T_{AB}$ and $T_{AC}$ if we know the angles $\theta$ and $\phi$. From the law of cosines

$$\theta = \cos^{-1}\left(\frac{D^2 + L_{AB}^2 - L_{AC}^2}{2DL_{AB}}\right)$$

From the law of sines

$$\phi = \sin^{-1}\left(\frac{L_{AB}\sin\theta}{L_{AC}}\right)$$

For the given values $D = 6$ ft, $L_{AB} = 3$ ft, and $W = 2000$ lb, use a loop in MATLAB to find $L_{AC\,\mathrm{min}}$, the shortest length $L_{AC}$ we can use without $T_{AB}$ or $T_{AC}$ exceeding 2000 lb. Note that the largest $L_{AC}$ can be is 6.7 ft (which corresponds to $\theta = 90°$). Plot the tension forces $T_{AB}$ and $T_{AC}$ on the same graph versus $L_{AC}$ for $L_{AC\,\mathrm{min}} \le L_{AC} \le 6.7$.

**30.*** In the structure in Figure P30a, six wires support three beams. Wires 1 and 2 can support no more than 1200 N each, wires 3 and 4 can support no more than 400 N each, and wires 5 and 6 no more than 200 N each. Three equal weights $W$ are attached at the points shown. Assuming that the structure is stationary and that the weights of the wires and the beams are very small compared to $W$, the principles of statics applied to a particular beam state that the sum of vertical forces is zero and that the sum of moments about any point is also zero. Applying these principles to each beam using the free-body diagrams shown in Figure P30b, we obtain the following equations. Let the tension force in wire $i$ be $T_i$. For beam 1

$$T_1 + T_2 = T_3 + T_4 + W + T_6$$

$$-T_3 - 4T_4 - 5W - 6T_6 + 7T_2 = 0$$