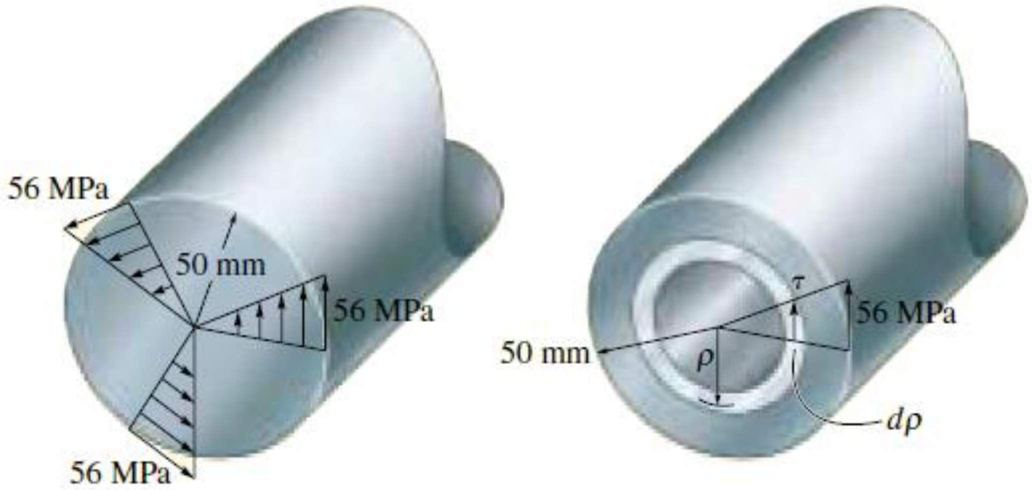
TUTORIAL- 5: Torsional Stress

PROBLEM-1



The stress distribution in a solid shaft has been plotted along three arbitrary radial lines as shown in Figure. Determine the resultant internal torque at the section.

PROBLEM-1

The polar moment of inertia for the cross-sectional area is

$$J = \frac{\pi}{2}(50\text{ mm})^4 = 9.82 \times 10^6\text{ mm}^4$$

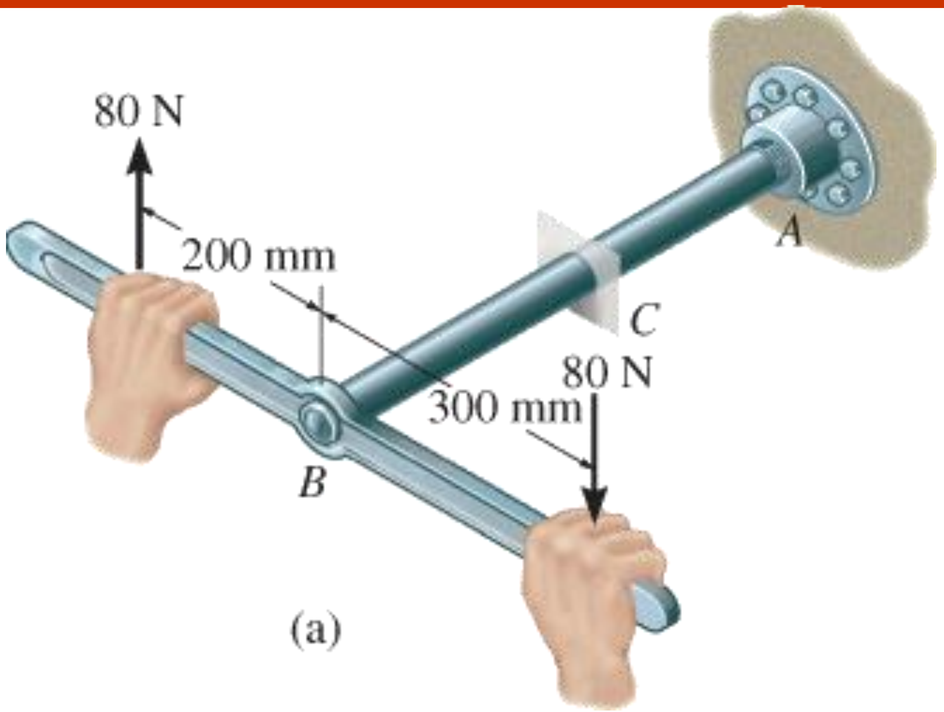
Applying the torsion formula, with $\tau_{max} = 56\text{ MPa} = 56\text{ N/mm}^2$, Fig. we have

$$\tau_{max} = \frac{Tc}{J}; \qquad 56\text{ N/mm}^2 = \frac{T\,(50\text{ mm})}{(9.82 \times 10^6)\text{ mm}^4}$$

$$T = 11.0\text{ kN}\cdot\text{m}$$

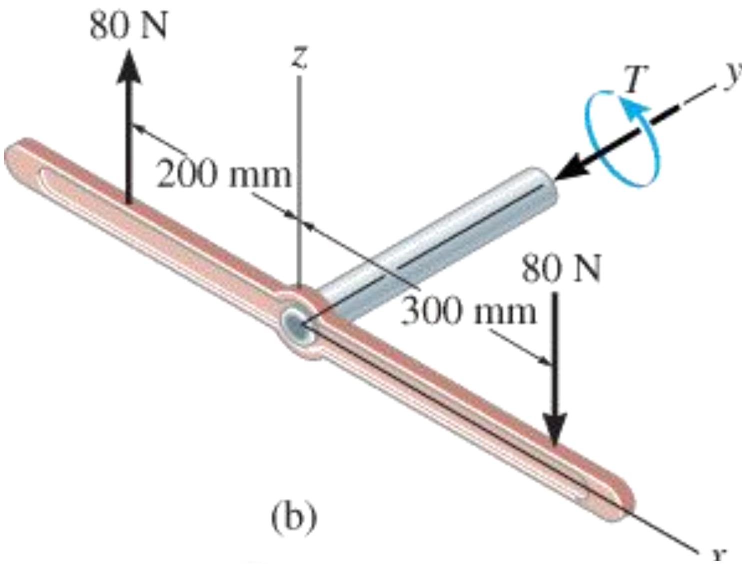
TUTORIAL- 5: Torsional Stress

PROBLEM-2

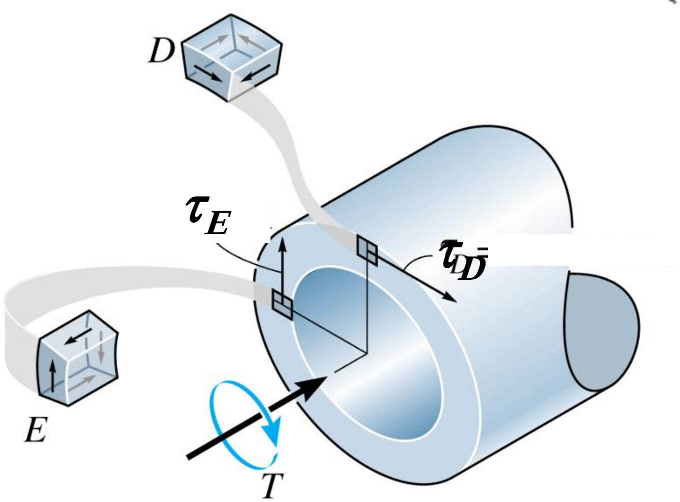


The pipe shown in the figure has an inner diameter of 80 mm and an outer diameter of 100 mm. If its end is tightened against the support at *A* using a torque wrench at *B*. Determine the shear stress developed in the material at the inner and outer walls along the central portion of the pipe when the 80-N forces are applied to the wrench.

PROBLEM-2



(b)



(c)

Torsion along the pipe is calculated by considering intermediate location *C*.

$T - 80(200) - 80(300) = 0$

$\Sigma M_y = 0,\quad T = 40\text{x}10^3\ \text{N.mm} = 40\ \text{N.m}$

Polar moment of inertia:

$$J = \frac{\pi}{2}\left(c_o^4 - c_i^4\right) = \frac{\pi}{2}\left[(50)^4 - (40)^4\right]\ \text{mm}^4$$

$$= 5.8\text{x}10^{-6}\ \text{m}^4$$

Shear stress in the outer surface:

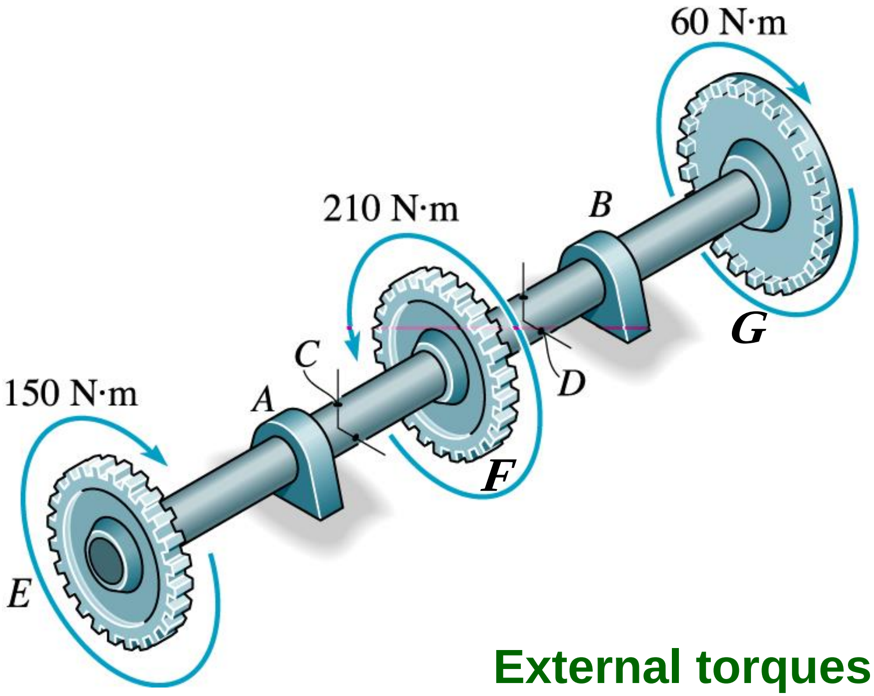
$$\tau_D = \frac{T c_o}{J} = \frac{(40)(0.05)}{5.8\text{x}10^{-6}} = 0.345\ \text{MPa}$$

Shear stress in the inner surface:

$$\tau_E = \frac{T c_i}{J} = \frac{(40)(0.04)}{5.8\text{x}10^{-6}} = 0.276\ \text{MPa}$$

TUTORIAL- 5: Torsional Stress

PROBLEM-3



The shaft has a diameter of 25 mm and it is subjected to the applied torques as shown. Determine the shear stress maximum along segments *EF* and *FG,* say that it is at point *C* and *D*.

External torques acting at each gears:

T_E = 150 N.m

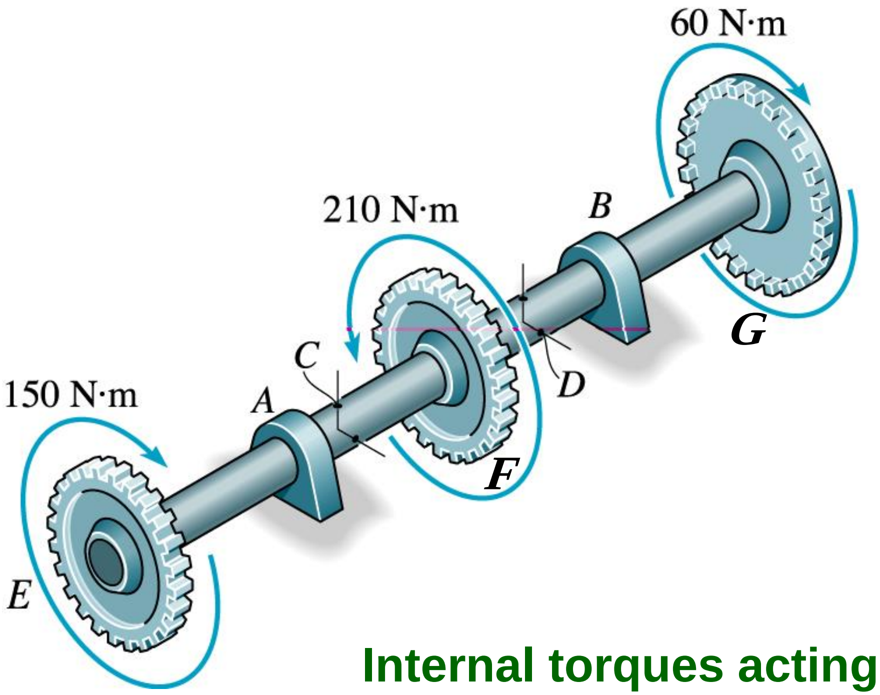
T_F = 210 N.m

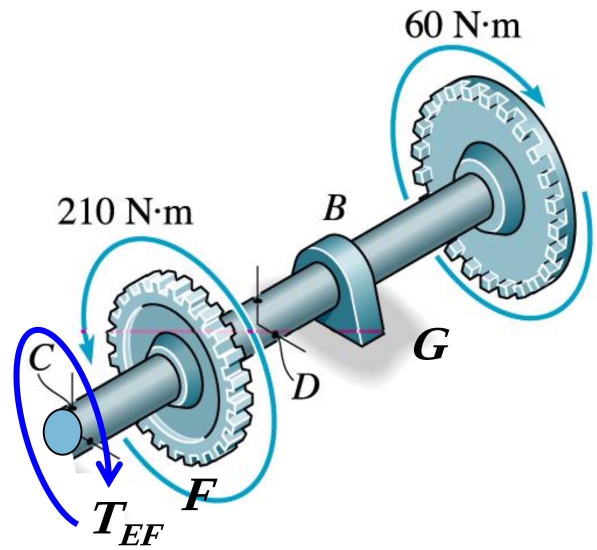
T_G = 60 N.m

Polar moment of inertia

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (12.5)^4 = 38.35\text{x}10^3 \text{ mm}^4 = 38.35\text{x}10^{-9} \text{ m}^4$$

PROBLEM-3





Internal torques acting along segment EF:

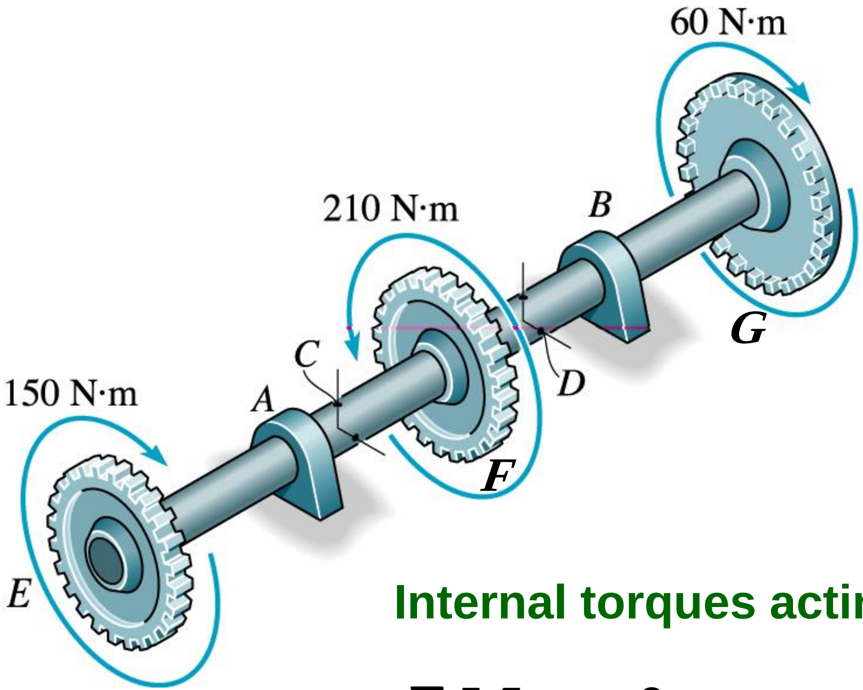
$$\Sigma M_{EF} = 0, \quad -T_{EF} + T_F - T_G = 0, \quad -T_{EF} + 210 - 60 = 0,$$

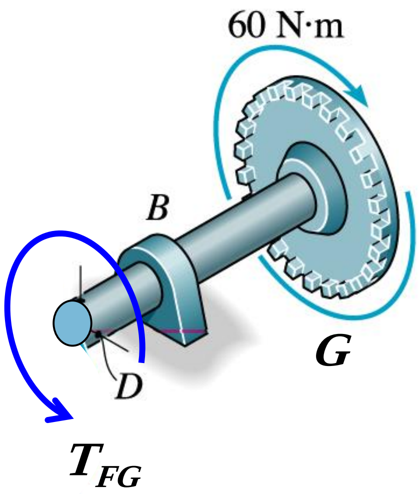
$$T_{EF} = 150 \text{ N.m}$$

Shear stress maximum along segment EF:

$$\tau_C = \tau_{max} = \frac{T_{EF}\, c}{J} = \frac{(150)(0.0125)}{38.35\text{x}10^{-9}} = 48.9\text{x}10^{6} \text{ N/m}^2 = 48.9 \text{ MPa}$$

PROBLEM-3





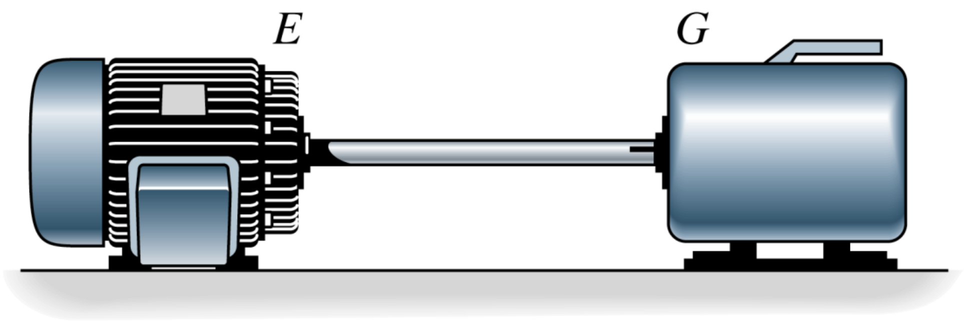
Internal torques acting along segment *FG*:

$$\Sigma M_{FG} = 0, \qquad T_{FG} - T_G = 0, \qquad T_{FG} = T_G = 60 \text{ N.m}$$

Shear stress maximum along segment *FG*:

$$\tau_D = \tau_{max} = \frac{T_{FG}\, c}{J} = \frac{(60)(0.0125)}{38.35\text{x}10^{-9}} = 19.55\text{x}10^{6} \text{ N/m}^2 = 19.55 \text{ MPa}$$

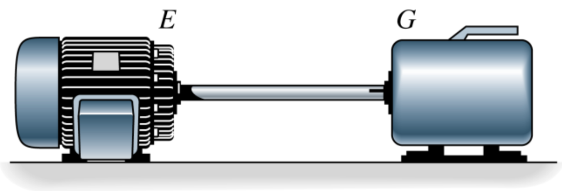
PROBLEM-4



The solid steel shaft is 3 m long and has a diameter of 50 mm. It is required to transmit 35 kW of power from the engine E to the generator G. Shear modulus of elasticity for the shaft material is 75 GPa. Determine:

1. **The smallest speed of the shaft in rpm if it is restricted not to twist more than 1^o.**
2. **Shear stress maximum in the shaft.**

PROBLEM-4



Polar moment of inertia

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (25)^4 = 613.6\text{x}10^3 \text{ mm}^4 = 613.6\text{x}10^{-9} \text{ m}^4$$

Maximum angle of twist: $\phi = (\pi/180^o)(1^o) = 0.01745$ rad

Allowable torque: $\phi = \dfrac{TL}{JG} \Rightarrow T = \dfrac{JG\phi}{L} = \dfrac{(613.6\text{x}10^{-9})(75\text{x}10^9)(0.01745)}{3} = 267.7$ N.m

Angular velocity: $P = T\omega \rightarrow \omega = P/T = \dfrac{(35\text{x}10^3)}{(267.7)} =$ 130.74 rad/sec

1. Maximum speed: $N = \dfrac{60 * 130.74}{2\pi} =$ 1248.5 rpm

2. Maximum shear stress: $\tau_{max} = \dfrac{Tc}{J} = \dfrac{(267.7)(0.025)}{613.6\text{x}10^{-9}} = 10.9\text{x}10^6 \text{ N/m}^2 = 10.9 \text{ MPa}$

PROBLEM-5

A tubular shaft, having an inner diameter of 30 mm and an outer diameter of 42 mm, is to be used to transmit 90 kW of power. Determine the frequency of rotation of the shaft so that the shear stress will not exceed 50 MPa.

The maximum torque that can be applied to the shaft is determined from the torsion formula.

$$\tau_{\max} = \frac{Tc}{J}$$

$$50(10^6)\ \text{N/m}^2 = \frac{T(0.021\ \text{m})}{(\pi/2)[(0.021\ \text{m})^4 - (0.015\ \text{m})^4]}$$

$$T = 538\ \text{N}\cdot\text{m}$$

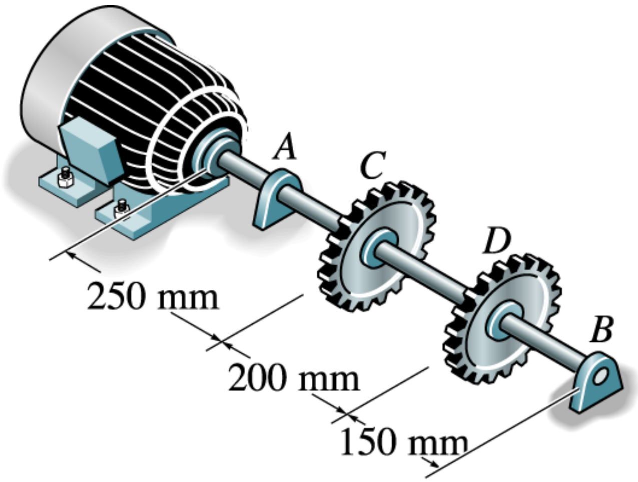
Applying Eq. 5–11, the frequency of rotation is

$$P = 2\pi f T$$

$$90(10^3)\ \text{N}\cdot\text{m/s} = 2\pi f(538\ \text{N}\cdot\text{m})$$

$$f = 26.6\ \text{Hz}$$

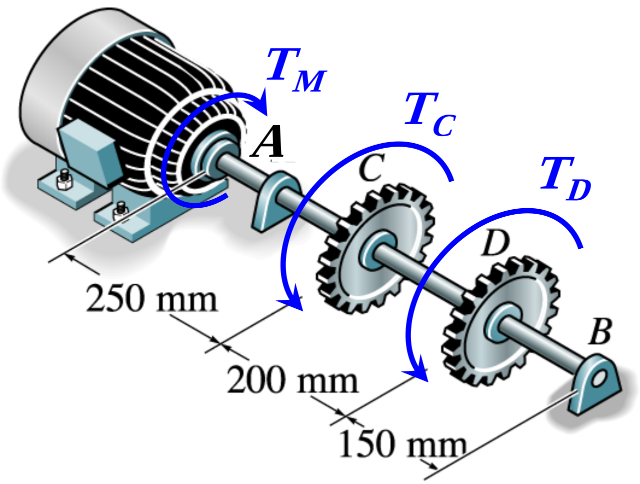
PROBLEM-6



The motor delivers 30 kW to the 304 stainless steel shaft while it rotates at 20 Hz. The shaft is supported on smooth bearings at A and B, which allow free rotation of the shaft. The gears C and D fixed to the shaft remove 18 kW and 12 kW, respectively.

The allowable stress is τ_{allow}= 56 MPa, the allowable angle of twist of C with respect to D is 0.20º, and shear modulus is G = 76 GPa. Determine the diameter of the shaft to the nearest mm.

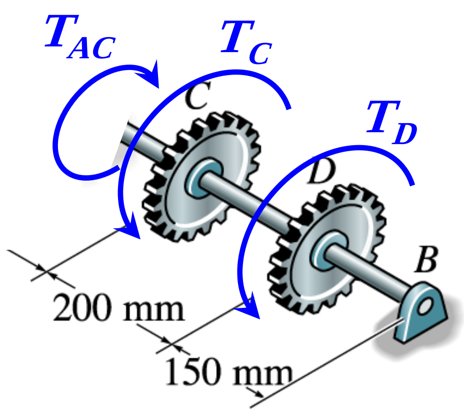
PROBLEM-6



External applied torque:

$$T_C = \frac{P_C}{2\pi f} = \frac{18x10^3}{2\pi(20)} = 143.24 \text{ N.m}$$

$$T_D = \frac{P_D}{2\pi f} = \frac{12x10^3}{2\pi(20)} = 95.49 \text{ N.m}$$



Internal torque developed in each segment:

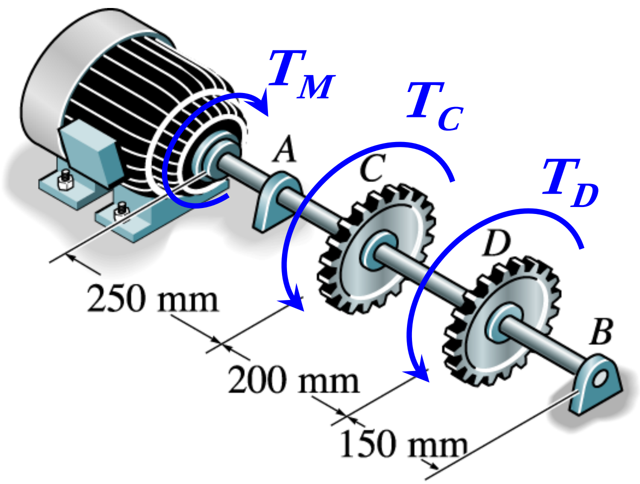
Segment *AC*:

$$T_{AC} = T_C + T_D = 238.73 \text{ N.m}$$

Segment *CD*:

$$T_{CD} = T_D = 95.49 \text{ N.m}$$

PROBLEM-6



Considering the failure due to the maximum shear stress

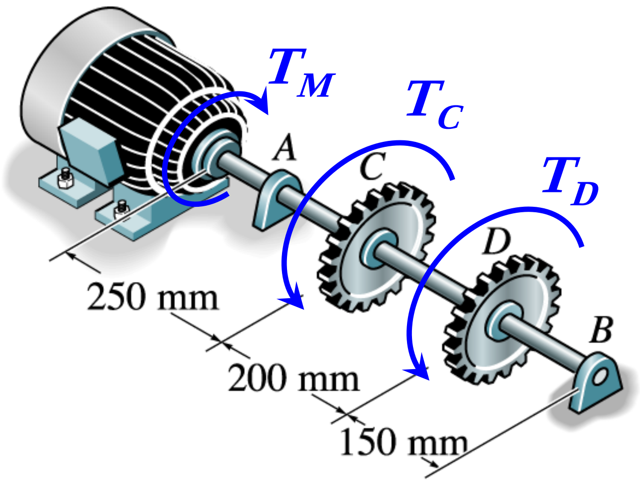
The critical segment is segment AC. Thus,

$$\tau_{max} = \tau_{allow} = \frac{T_{AC}\,r}{J} = \frac{T_{AC}\,(d/2)}{\frac{\pi}{32} d^4}$$

$$56\text{x}10^6 = \frac{238.73(d/2)}{\frac{\pi}{32} d^4}$$

$$d = 27.9\text{x}10^{-3}\text{ m} = 27.9\text{ mm}$$

PROBLEM-6



Considering the failure due to angle of twist limitation at segment *CD*

$$\phi_{C/D} = \frac{T_{CD} L_{CD}}{JG} = \frac{T_{CD} L_{CD}}{\left(\frac{\pi}{32} d^4\right) G}$$

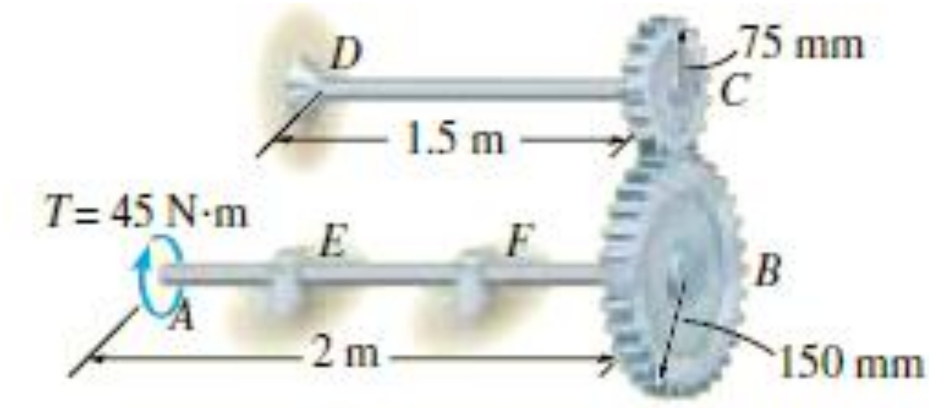
$$0.20\left(\frac{\pi}{180}\right) = \frac{95.49\left(200\text{x}10^{-3}\right)}{\left(\frac{\pi}{32} d^4\right) 76\text{x}10^9}$$

$$d = 27.2\text{x}10^{-3}\ \text{m} = 27.2\ \text{mm}$$

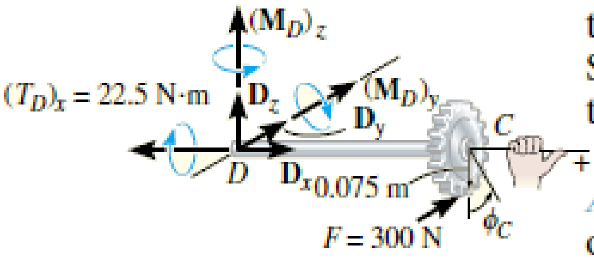
From both values of diameter, the required diameter is 27.9 mm, and we use *d* = 28 mm

PROBLEM-7

The two solid steel shafts shown in Figure are coupled together using the meshed gears. Determine the angle of twist of end A of shaft AB when the torque T = 45 N.m is applied. Take G = 80 GPa. Shaft AB is free to rotate within bearings E and F, whereas shaft DC is fixed at D. Each shaft has a diameter of 20 mm.



PROBLEM-7

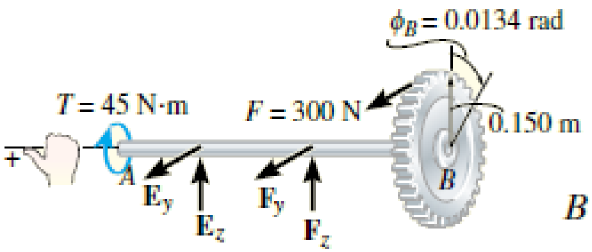


Internal Torque. Free-body diagrams for each shaft are shown in Fig. 5–21*b* and 5–21*c*. Summing moments along the *x* axis of shaft *AB* yields the tangential reaction between the gears of $F = 45\ \text{N}\cdot\text{m}/0.15\ \text{m} = 300\ \text{N}$. Summing moments about the *x* axis of shaft *DC*, this force then creates a torque of $(T_D)_x = 300\ \text{N}(0.075\ \text{m}) = 22.5\ \text{N}\cdot\text{m}$ on shaft *DC*.

Angle of Twist. To solve the problem, we will first calculate the rotation of gear *C* due to the torque of 22.5 N · m in shaft *DC*, Fig. 5–21*b*. This angle of twist is

$$\phi_C = \frac{TL_{DC}}{JG} = \frac{(+22.5\ \text{N}\cdot\text{m})(1.5\ \text{m})}{(\pi/2)(0.010\ \text{m})^4[80(10^9)\ \text{N/m}^2]} = +0.0269\ \text{rad}$$

Since the gears at the end of the shaft are in mesh, the rotation ϕ_C of gear *C* causes gear *B* to rotate ϕ_B, Fig. 5–21*c*, where



$$\phi_B(0.15\ \text{m}) = (0.0269\ \text{rad})(0.075\ \text{m})$$
$$\phi_B = 0.0134\ \text{rad}$$

We will now determine the angle of twist of end *A* with respect to end *B* of shaft *AB* caused by the 45 N · m torque, Fig. 5–21*c*. We have

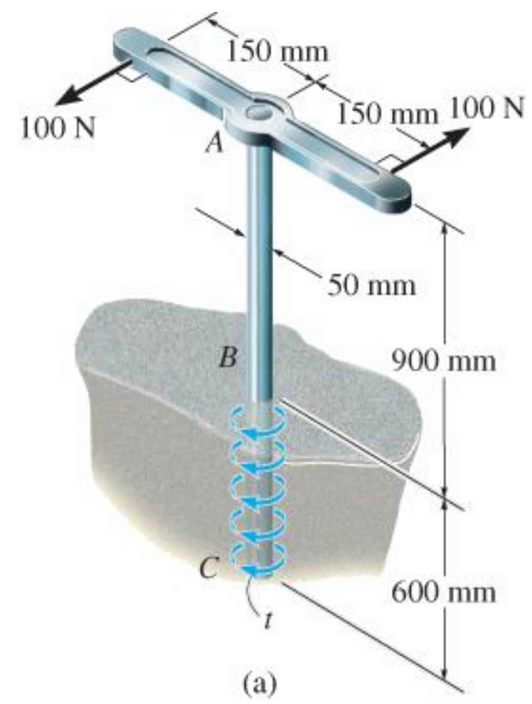
$$\phi_{A/B} = \frac{T_{AB}L_{AB}}{JG} = \frac{(+45\ \text{N}\cdot\text{m})(2\ \text{m})}{(\pi/2)(0.010\ \text{m})^4[80(10^9)\ \text{N/m}^2]} = +0.0716\ \text{rad}$$

The rotation of end *A* is therefore determined by adding ϕ_B and $\phi_{A/B}$, since both angles are in the *same direction*, Fig. 5–21*c*. We have

$$\phi_A = \phi_B + \phi_{A/B} = 0.0134\ \text{rad} + 0.0716\ \text{rad} = +0.0850\ \text{rad} \quad \textit{Ans.}$$

PROBLEM-8

50-mm-diameter solid cast-iron post shown is buried 600 mm in soil. Determine maximum shear stress in the post and angle of twist at its top. Assume torque about to turn the post, and soil exerts uniform torsional resistance of t N·mm/mm along its 600 mm buried length. $G = 40(10^3)$ MPa

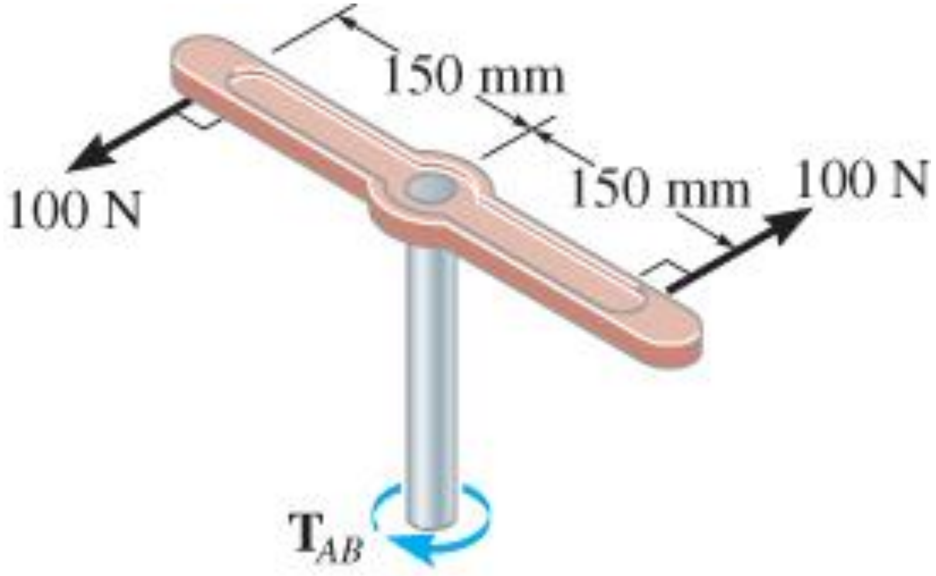


PROBLEM-8

Internal torque

From free-body diagram

$\Sigma M_z = 0; \qquad T_{AB} = 100\ \text{N}(300\ \text{mm}) = 30 \times 10^3\ \text{N}\cdot\text{mm}$

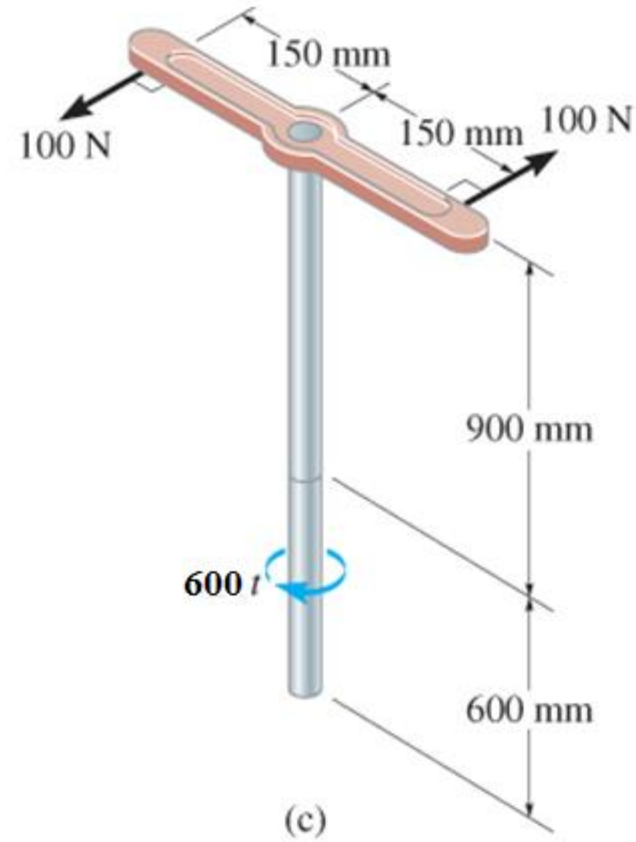


(b)

PROBLEM-8

Internal torque

Magnitude of the uniform distribution of torque along buried segment BC can be determined from equilibrium of the entire post.



$$\Sigma M_z = 0;$$

$$100\text{ N}(300\text{ mm}) - t(600\text{ mm}) = 0$$

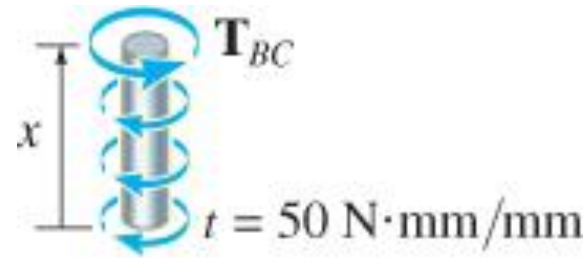
$$t = 50\text{ N}\cdot\text{mm/mm}$$

PROBLEM-8

Internal torque

Hence, from free-body diagram of a section of the post located at position x within region BC, we have

$$\Sigma M_z = 0;$$



(d)

$$T_{BC} - 50x = 0$$

$$T_{BC} = 50x$$

PROBLEM-8

Maximum shear stress

Largest shear stress occurs in region AB, since torque largest there and J is constant for the post. Applying torsion formula

$$\tau_{max} = \frac{T_{AB}\,c}{J} = \ldots = 1.22\ \mathrm{N/mm^2}$$

PROBLEM-8

Angle of twist

Angle of twist at the top can be determined relative to the bottom of the post, since it is fixed and yet is about to turn. Both segments AB and BC twist, so

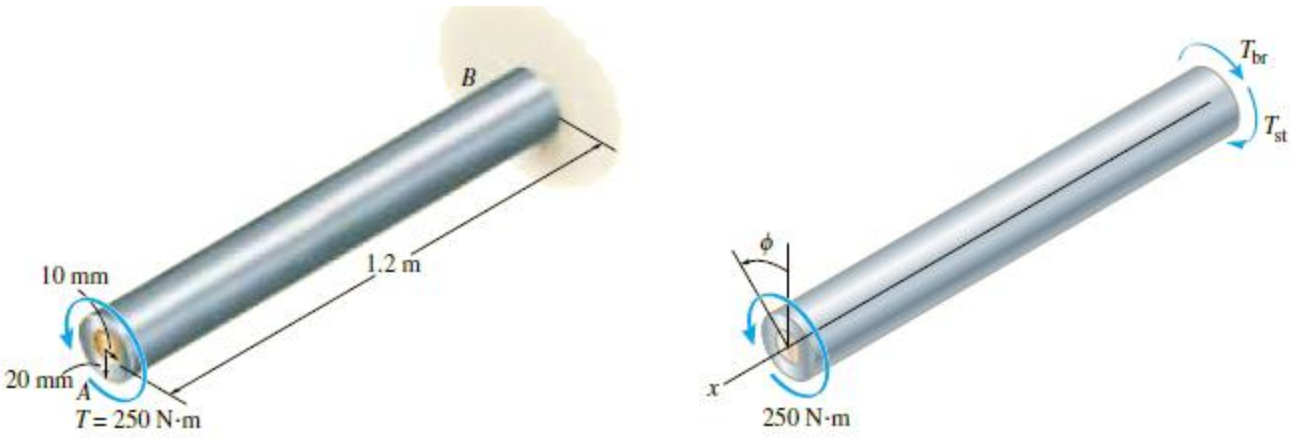
$$\phi_A = \frac{T_{AB} L_{AB}}{JG} + \int_0^{L_{BC}} \frac{T_{BC}\, dx}{JG}$$

$$\vdots$$

$$\phi_A = 0.00147 \text{ rad}$$

PROBLEM-9

The shaft shown in Figure is made from a steel tube, which is bonded to a brass core. If a torque of T = 250 N·m is applied at its end, plot the shear-stress distribution along a radial line of its cross-sectional area. Take G_{st} = 80 GPa, G_{br} = 36 GPa.



PROBLEM-9

Equilibrium. A free-body diagram of the shaft is shown in Fig. 5–26*b*. The reaction at the wall has been represented by the unknown amount of torque resisted by the steel, T_{st}, and by the brass, T_{br}. Equilibrium requires

$$-T_{st} - T_{br} + 250 \text{ N·m} = 0 \qquad (1)$$

Compatibility. We require the angle of twist of end A to be the same for both the steel and brass since they are bonded together. Thus,

$$\phi = \phi_{st} = \phi_{br}$$

Applying the load–displacement relationship, $\phi = TL/JG$, we have

$$\frac{T_{st}L}{(\pi/2)[(20 \text{ mm})^4 - (10 \text{ mm})^4]80(10^3) \text{ N/mm}^2} =$$

$$\frac{T_{br}L}{(\pi/2)(10 \text{ mm})^4 36(10^3) \text{ N/mm}^2}$$

$$T_{st} = 33.33\, T_{br} \qquad (2)$$

PROBLEM-9

Solving Eqs. 1 and 2, we get

$$T_{st} = 242.72\ \text{N}\cdot\text{m}$$

$$T_{br} = 7.28\ \text{N}\cdot\text{m}$$

These torques act throughout the entire length of the shaft, since no external torques act at intermediate points along the shaft's axis. The shear stress in the brass core varies from zero at its center to a maximum at the interface where it contacts the steel tube. Using the torsion formula,

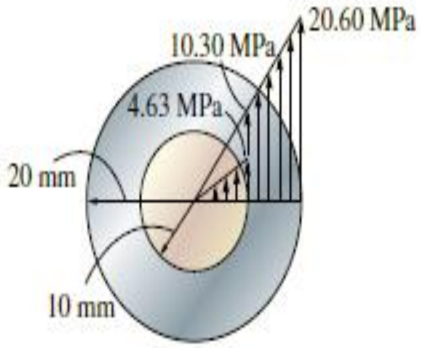
$$(\tau_{br})_{max} = \frac{7.28\ \text{N}\cdot\text{mm}\cdot(10^3)\ \text{mm/m}\cdot 10\ \text{mm}}{(\pi/2)(10\ \text{mm})^4} = 4.63\ \text{N/mm}^2 = 4.63\ \text{MPa}$$

For the steel, the minimum shear stress is also at this interface,

$$(\tau_{st})_{min} = \frac{242.72\ \text{N}\cdot\text{m}\cdot 10^3\ \text{mm/m}\cdot 10\ \text{mm}}{(\pi/2)[(20\ \text{mm})^4 - (10\ \text{mm})^4]} = 10.30\ \text{N/mm}^2 = 10.30\ \text{MPa}$$

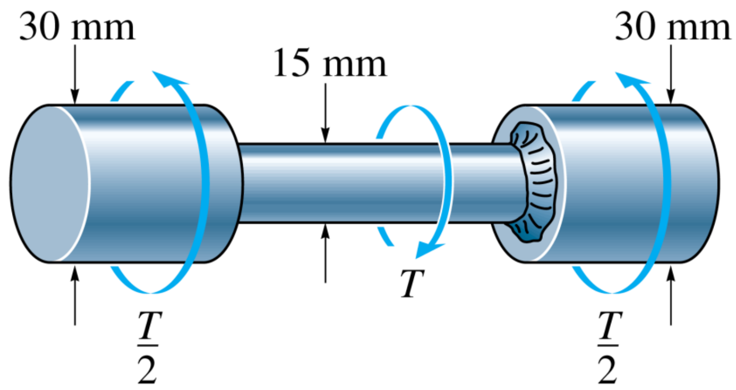
and the maximum shear stress is at the outer surface,

$$(\tau_{st})_{max} = \frac{242.72\ \text{N}\cdot\text{m}\cdot 10^3\ \text{mm/m}\cdot 20\ \text{mm}}{(\pi/2)[(20\ \text{mm})^4 - (10\ \text{mm})^4]} = 20.60\ \text{N/mm}^2 = 20.60\ \text{MPa}$$



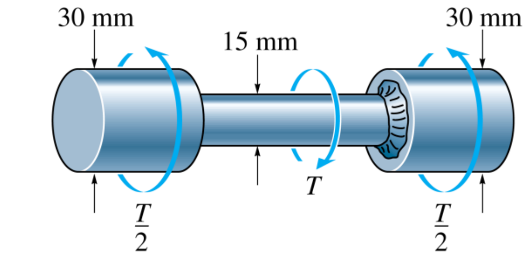
Shear–stress distribution

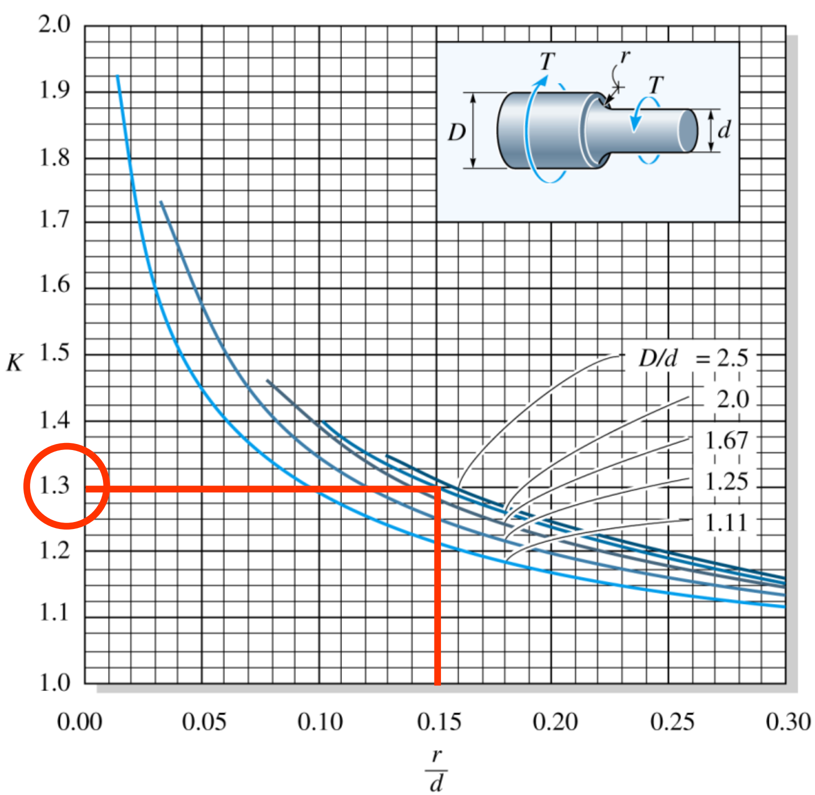
PROBLEM-10



The steel used for the shaft has an allowable shear stress of τ_{allow}= 8 MPa. If the members are connected together with a fillet weld of radius r = 2.25 mm, determine the maximum torque T that can be applied.

PROBLEM-10





Stress concentration factor

$$\frac{r}{d} = \frac{2.25}{15} = 0.15 \qquad \frac{D}{d} = \frac{30}{15} = 2$$

Referring to the graph: $K = 1.3$

Nominal shear stress $$\tau_{nom} = \frac{\left(\frac{T}{2}\right)c}{J}$$

Maximum shear stress

$$\tau_{max} = \tau_{allow} = K\,\tau_{nom}$$

Rearranging the equation, we have

$$\left(\frac{T}{2}\right) = \frac{J\,\tau_{allow}}{K\,c} \quad \rightarrow \quad T = 8.2\text{ N.m}$$