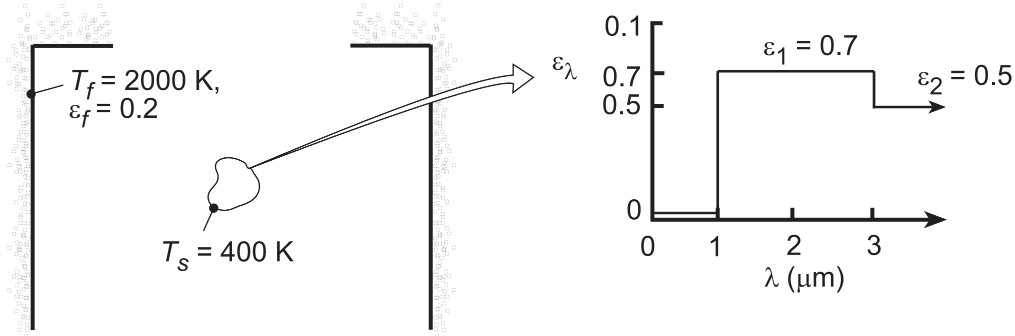
PROBLEM 12.50

KNOWN: Temperature and spectral emissivity of small object suspended in large furnace of prescribed temperature and total emissivity.

FIND: (a) Total surface emissivity and absorptivity, (b) Reflected radiative flux and net radiative flux to surface, (c) Spectral emissive power at $\lambda = 2\ \mu m$, (d) Wavelength $\lambda_{1/2}$ for which one-half of total emissive power is in spectral region $\lambda \geq \lambda_{1/2}$.

SCHEMATIC:



ASSUMPTIONS: (1) Surface is opaque and diffuse, (2) Walls of furnace are much larger than object.

ANALYSIS: (a) The emissivity of the object may be obtained from Eq. 12.43, which is expressed as

$$\varepsilon(T_s) = \frac{\int_0^\infty \varepsilon_\lambda(\lambda) E_{\lambda,b}(\lambda, T_s)\, d\lambda}{E_b(T)} = \varepsilon_1\left[F_{(0\to 3\mu m)} - F_{(0\to 1\mu m)}\right] + \varepsilon_2\left[1 - F_{(0\to 3\mu m)}\right]$$

where, with $\lambda_1 T_s = 400\ \mu m\cdot K$ and $\lambda_2 T_s = 1200\ \mu m\cdot K$, $F_{(0\to 1\mu m)} = 0$ and $F_{(0\to 3\mu m)} = 0.002$. Hence,

$$\varepsilon(T_s) = 0.7(0.002) + 0.5(0.998) = 0.500$$ <

The absorptivity of the surface is determined by Eq. 12.52,

$$\alpha = \frac{\int_0^\infty \alpha_\lambda(\lambda) G_\lambda(\lambda)\, d\lambda}{\int_0^\infty G_\lambda(\lambda)\, d\lambda} = \frac{\int_0^\infty \alpha_\lambda(\lambda) E_{\lambda,b}(\lambda, T_f)\, d\lambda}{E_b(T_f)}$$

Hence, with $\lambda_1 T_f = 2000\ \mu m\cdot K$ and $\lambda_2 T_f = 6000\ \mu m\cdot K$, $F_{(0\to 1\mu m)} = 0.067$ and $F_{(0\to 3\mu m)} = 0.738$. It follows that

$$\alpha = \alpha_1\left[F_{(0\to 3\mu m)} - F_{(0\to 1\mu m)}\right] + \alpha_2\left[1 - F_{(0\to 3\mu m)}\right] = 0.7\times 0.671 + 0.5\times 0.262 = 0.601$$ <

(b) The reflected radiative flux is

$$G_{ref} = \rho G = (1-\alpha) E_b(T_f) = 0.399\times 5.67\times 10^{-8}\ W/m^2\cdot K^4\ (2000\ K)^4 = 3.620\times 10^5\ W/m^2$$ <

The net radiative flux to the surface is

$$q''_{rad} = G - \rho G - \varepsilon E_b(T_s) = \alpha E_b(T_f) - \varepsilon E_b(T_s)$$

$$q''_{rad} = 5.67\times 10^{-8}\ W/m^2\cdot K^4\left[0.601(2000\ K)^4 - 0.500(400\ K)^4\right] = 5.438\times 10^5\ W/m^2$$ <

(c) At $\lambda = 2\ \mu m$, $\lambda T_s = 800$ K and, from Table 12.1, $I_{\lambda,b}(\lambda,T)/\sigma T^5 = 0.991\times 10^{-7}\ (\mu m\cdot K\cdot sr)^{-1}$. Hence,

Continued...

PROBLEM 12.50 (Cont.)

$$I_{\lambda,b} = 0.991\times10^{-7}\times5.67\times10^{-8}\,\frac{W/m^2\cdot K^4}{\mu m\cdot K\cdot sr}\times(400\,K)^5 = 0.0575\,\frac{W}{m^2\cdot\mu m\cdot sr}$$

Hence, with $E_\lambda = \varepsilon_\lambda E_{\lambda,b} = \varepsilon_\lambda \pi I_{\lambda,b}$,

$$E_\lambda = 0.7(\pi sr)\,0.0575\,W/m^2\cdot\mu m\cdot sr = 0.126\,W/m^2\cdot\mu m$$ <

(d) From Table 12.1, $F_{(0\to\lambda)} = 0.5$ corresponds to $\lambda T_s \approx 4100$ μm·K, in which case,

$$\lambda_{1/2} \approx 4100\,\mu m\cdot K/400\,K \approx 10.3\,\mu m$$ <

COMMENTS: Because of the significant difference between T_f and T_s, $\alpha \neq \varepsilon$. With increasing $T_s \to T_f$, ε would increase and approach a value of 0.601.